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New Optimal AH Method for Solving Transportation Problems

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Abstract

The main purpose of the paper is to develop a direct, new optimal method to minimize the total transportation cost from origin to destination by meeting supply and demand limits. Hundreds of transportation problems from different research papers have been solved by a new proposed optimal method. Its results are compared with the well-known and established techniques, especially the Steeping Stone method (1954) and the Modified distribution (MODI) method (1989), and it is found that all results obtained by the proposed method are optimal. In this paper, we have proposed a new optimal method, called the AH method for solving transportation problems. In the proposed method, there is no need to find an initial basic feasible solution (IBFS) because this is a direct method to find the optimal solution. Whenever the Steeping Stone method and the MODI method depend upon an initial feasible solution because both are not direct methods. A software package based on Java script programming designed by Html, CSS and Bootstrap was developed for new optimal AH method. Hundreds of balanced and unbalanced problems of different sizes solved from different research papers by using the software that provided optimal results.

Keywords: *Transportation problems, Initial Basic Feasible Solution (IBFS) methods, MODI and Stepping Stone methods, New AH optimal Method, Cost comparison and verification*.

1. INTRODUCTION

Transportation problems (TP) are a crucial component of operations research, focusing on the optimal distribution of resources from multiple suppliers to various consumers while minimizing costs. TP is a unique variant of the Linear Programming Problem (LPP) mainly used for reducing transportation costs. The initial substantive study on this issue was carried out by Hitchcock in 1941, titled as, "The Distribution of a Product from Several Sources to Numerous Localities". (Hitchcock et al., 1941)[10]. This was the inaugural formal contribution to transportation-related problems. Gleyzal (1955)[9] the transportation problem is called for any pioneer application including several product resources and several activities or destinations of products. The method deals with applications of problems involving transporting goods or products from several sources to several destinations that is why it named transportation problem. Although the model can be used to distribution and transportation problems as well as represent more general scheduling and assignment problems. Maximize the profit of transport m unities to n destinations or minimize the cost of transport m unities to n destinations are the two collective objectives of such problems. The TP has two stages for obtaining the optimal solution. The Initial Basic Feasible Solution (IBFS) is the first stage, which aims to find an initial solution with minimal total cost. These methods include North West Corner Method (NWCM) (Charnes et al., 1953) [5] developed. It takes Very first cell (North West Cost cell) in the transportation table and allocates minimum supply demand (S, D) to that cell and calculates total cost. Later, another IBFS method Least Cost Method (LCM) developed (Dantzig et al., 1963) [7]. It takes minimum cell in the Transportation table and allocates minimum supply demand (S, D) to that cell and calculates total cost. After that some of the well-received heuristic methods were developed, including the Vogel's Approximation Method (VAM), which penalizes inconsistency by measuring the difference between the lowest and second lowest costs in each row and column of a transportation matrix, assigning the least cost cell the highest penalty (Reinfeld & Vogel, 1958) [23]. Improved form of ATCM (Malvia et al., 2018) [19] introduced. It finds penalty of each row and column which is equal to the average of two minimum costs in the row/column. Takes maximum penalty and uses min:(S, D) to calculate total cost. It provides IBFS. Direct approach method (FCM) was composed. It Subtracts largest cell cost from the sum of two smallest cell costs. Then takes maximum cost with row or column, uses min:(S, D) and finds total cost, it provides IBFS (Mohanta et al.,2019) [20]. The second stage involves finding the optimal solution.

Mainly there are two types of the methods for solving transportation problems.

I. Initial basic feasible solution methods

- North West Corner method [5].
- Allocation Table method [4].

- ABC method [1].
- Average Total Opportunity Cost method [3].
- Least cost method [7].
- Lowest Supply and Demand Method [11].
- ICVM method [15].
- RICM method [16].
- Direct Sum Method [18].
- Palsu's Favorable Cost Method [20].
- TOCM-MT method [21].
- A revised version of asm-method [22].
- Vogel's approximation method [23].
- An alternate method to matrix minima method [25] etc.

II. Optimal methods

- Modified Distribution Method (MODI) [2].
- Stepping Stone method [6].

In every research journal or in the text books of operations research mathematicians have used different methods to solve the transportation problems in two stages.

- In the first stage, the initial basic feasible solution (IBFS) was obtained by any available methods.
- In the second stage, once an initial feasible solution is obtained, the Modified Distribution Method (MODI) can be applied to optimize it further, refining allocations to minimize costs effectively.

In this paper, a new proposed method finds a direct optimal solution without finding an initial basic feasible solution through many numerical examples.

2. MATHEMATICAL MODEL FOR TRANSPORTATION PROBLEMS

Let $X_{ij} \geq 0$ be the quantity shipped from the source i to the destination j. The mathematical formulation of the problem is as follows:

Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$ (Total transportation cost)

Subject to $\sum_{j=1}^{n} X_{ij} = a_i$ (Supply from sources), i = 1,2,3,…..,m and j = 1,2,3,…..,n

$\sum_{i=1}^{m} X_{ij} = b_j$ (Demand from destination), i = 1,2,3,……,m and j = 1,2,3,...n

Where,

Z: Total transportation cost to be minimized.

C_{ij}: Unit transportation cost of the commodity from each source i to destination j.

X_{ij}: Number of units of commodity sent from source i to destination j and $X_{ij} \geq 0$ for all i and j.

a_i : the commodity available at each source i.

b_j: the commodity required on demand at each destination j.

Note: Transportation model is balanced if,

$$\text{Supply}\left(\sum_{i=1}^{m} a_i\right) = \text{Demand}\left(\sum_{i=1}^{m} b_i\right).$$

Otherwise, unbalanced if,

$$Supply\left(\sum_{i=1}^{m} a_i\right) \neq Demand\left(\sum_{i=1}^{m} b_i\right).$$

The total number of variables is *mxn*. The total number of constraints is *m+n*, while the total number of allocations (m+n–1) should be in feasible solution. Here the letter *m* denotes the number of rows and *n* denotes the number of columns.

2.1 THE PROPOSED METHOD

New Optimal AH Method for Solving Transportation Problems.

The idea of this method is taken from the Stepping Stone method which was developed in 1954 by A. Charnes a renowned mathematician [6]. The new AH optimal method provides direct optimal solution of the transportation problems. It is based on selecting least cost cell in each row and each column by allocating min: (S,D) to the selected cell/cells simultaneously where ever possible on the basis of first come first serve and delete entire row/column and calculate total transportation cost. This method obtains the optimal solution without finding an IBFS.

Algorithm.

Step 1. In a balanced model select least cost cell in each row and each column, allocate min:(S,D) to the selected cell/cells simultaneously where ever possible on the basis of first come first serve and delete entire row/column.

Step 2. If there are more than two selected cells in associated row and column then allocate min:(S,D) to the cell which contains minimum cost and delete entire row or column. Continue the process until the condition m+n-1 is satisfied for the occupied cells, where m is the number of rows and n is the number of columns. Calculate the total cost.

Step 3. Now take all unoccupied cells one by one and draw a loop that makes a right angle at occupied cells and returns back at original cell. Add the transportation cost of the cells of the traced path by using (+) and (-) signs alternatively. If net cost of the path of each unoccupied cell is ≥ 0, stop; the current solution is optimal, otherwise go to step 4.

Step 4. If net cost of the path of any cell/cells is < 0, that becomes occupied cell and the cell/cells in this path with minimum allocation at negative slot is/are now unoccupied. Hence the condition m+n-1 is satisfied for the occupied cells. Now take original model mark all the occupied cells, apply minimum (S,D) allocation to each one by one and calculate total minimum cost. Repeat the steps 3 and 4 until the solution is optimal.

2.2 NUMERICAL EXAMPLES

A software package based on Java script programming designed by Html, CSS and Bootstrap was developed for new optimal AH method (see fig.**1**, page no. **11**). Using Software, we have solved hundreds of numerical examples of different sizes balanced and unbalanced problems taken from several research journals and results were obtained optimal. These research journals are mentioned in the reference section. Two examples are solved, one by using software (see figures: **1, 2a, 2b, 2c, 3a** and **3b**) and other example solved manually stepwise (see tables **1**to **9-page** no. **6** to **10**) by using optimal AH method as follows:

Example. Using New Optimal AH Method to solve the following transportation problem.

Table 1.

	D_1	**D_2**	**D_3**	**Supply**
S_1	3	3	5	9
S_2	6	5	4	8
S_3	6	10	7	10
Demand	7	`12	8	27

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Solution: - Using algorithm (see section **2.1**, page no. **5**)

Table 2.

	D_1	D_2	D_3	Supply
S_1	**3**	**3**	5	9
S_2	6	5	**4**	8
S_3	**6**	10	7	10
Demand	7	`12	8	27

Step 1. In a balanced model select number of least cost cells from each row, each column and allocate min: (S,D) to the selected cell/cells simultaneously where ever possible on the basis of first come first serve and delete entire row/column.

So here selected least cells in each row and column are, C_{11}**= 3,** C_{12}**=3,** C_{23}**=4** and C_{31}**=6**.

Table

	D_1	D_2	D_3	Supply
S_1	**3**	**3**	5	9
S_2	6	5	**4(8)**	8
S_3	**6**	10	7	10
Demand	7	`12	8	27

Table 4.

	D_1	D_2	D_3	Supply
S_1	**3**	**3(9)**	5	9
S_3	**6**	10	7	10
Demand	7	`12	0	27

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Step 2. If there are more than two selected cells in associated row and column then allocate min:(S,D) to the cell which contains minimum cost and delete entire row or column. So here **C_{11}(3)** and **C_{12}(3)** are the selected cell which contain minimum cost. Take **C_{12}(3)** for allocation.

Table 5.

	D_1	**D_2**	**D_3**	**Supply**
S_3	**6(7)**	10(3)	7(0)	10
Demand	7	`3	0	27

Finally we have single row so allocate min:(S,D) to all cell cost and final table is:

Table 6.

	D_1	**D_2**	**D_3**	**Supply**
S_1	3	3(9)	5	9
S_2	6	5	4(8)	8
S_3	6(7)	10(3)	7(0)	10
Demand	7	`12	8	27

To check Optimality condition we use, **step 3**.

Step 3. Now take all unoccupied cells one by one and draw a loop that makes a right angle at occupied cells and returns back at original cell. Add the cost of the cells of the traced path by using (+) and (-) signs alternatively. If net cost of the path of each unoccupied cell is ≥ 0, stop; the current solution is optimal, otherwise go to **step 4**.

To check whether the solution is optimal or not we take all unoccupied cells from above table.

These are **C_{11} (3), C_{13} (5), C_{21} (6) and C_{22} (5)**

1. Path for **C_{11} (3):**

$C_{11} \longrightarrow C_{12} \longrightarrow C_{32} \longrightarrow C_{31} = 3- 3+10-6 = 4 \geq 0$

2. Path for **C_{13} (5)**:

$C_{13} \longrightarrow C_{12} \longrightarrow C_{32} \longrightarrow C_{33} = 5-3+10-7 = 5 \geq 0$

3. Path for **C_{21} (6)**:

International Journal of Supply Chain and Logistics
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Vol. 9, Issue No.3, pp 74 - 92, 2025
www.carijournals.org

$C_{21} \longrightarrow C_{23} \longrightarrow C_{33} \longrightarrow C_{31}$ = 6- 4+7-6 = 3 $\geq$ 0

4. Path for **C_{22} (5)**:

$C_{22} \longrightarrow C_{23} \longrightarrow C_{33} \longrightarrow C_{32}$ = 5-4+7-10= -2 < 0

Since path of one of the cell is < 0 so solution is not optimal, so go to **step 4**.

Step 4. If net cost of the path of any cell/cells is < 0, that becomes occupied cell and the cell/cells in this path with minimum allocation at negative slot is/are now unoccupied. Hence the condition **m+n-1** is satisfied for the occupied cells. Now take original model mark all the occupied cells, apply minimum (S,D) allocation to each one by one and calculate total minimum cost. Repeat the steps 3 and 4 until the solution is optimal.

Here cell **C_{22}(5)** has negative path so that becomes occupied cell and the cell in this path **C_{32}(10)** at negative slot with minimum allocation **3** is now unoccupied. So new table after marking is:

Table 7.

	D_1	**D_2**	**D_3**	**Supply**
S_1	3	**3**	5	9
S_2	6	**5**	4	8
S_3	**6**	**10**	**7**	10
Demand	7	`12	8	27

Again use **step 1**, allocate min:(S,D) where there is possible and we get new table as following

Table 8.

	D_1	**D_2**	**D_3**	**Supply**
S_1	3	**3(9)**	5	9
S_2	6	**5(3)**	4(5)	8
S_3	**6(7)**	**10**	**7(3)**	10
Demand	7	`12	8	27

To check Optimality condition we use, **step 3**.

Step 3. Now take all unoccupied cells one by one and draw a loop that makes a right angle at occupied cells and returns back at original cell. Add the transportation cost of the cells of the traced path by using (+) and (-) signs alternatively. If net cost of the path of each unoccupied cell is ≥ 0, stop; the current solution is optimal, otherwise go to **step 4**.

To check whether the solution is optimal or not we take all unoccupied cells from above table.

These are **C_{11} (3), C_{13} (5), C_{21} (6) and C_{32} (10),**

1. Path for **C_{11} (3)**:

$C_{11} \longrightarrow C_{12} \longrightarrow C_{22} \longrightarrow C_{23} \longrightarrow C_{33} \longrightarrow C_{31}$ = 3-3+5-4 + 7-6 = 2 $\geq$ 0

2. Path for **C_{13} (5)**:

$C_{13} \longrightarrow C_{12} \longrightarrow C_{22} \longrightarrow C_{23}$ = 5- 3+5-4 = 3 $\geq$ 0

3. Path for **C_{21} (6)**:

$C_{21} \longrightarrow C_{23} \longrightarrow C_{33} \longrightarrow C_{31}$ = 6-4+7- 6= 3 $\geq$ 0

4. Path for **C_{32} (10):**

$C_{32} \longrightarrow C_{33} \longrightarrow C_{23} \longrightarrow C_{22}$ = 10- 7+4-5 = 2 $\geq$ 0.

Since the path of all unoccupied cells are **$\geq$ 0**, so the above solution is Optimal. Final table is:

Table 9.

	D_1	**D_2**	**D_3**	**Supply**
S_1	3	**3(9)**	5	9
S_2	6	**5(3)**	4(5)	8
S_3	**6(7)**	**10**	**7(3)**	10
Demand	7	`12	8	27

Hence the total cost,

Z= 3(9)+5(3)+4(5)+6(7)+7(3)= 27+15+20+42+21= 125, which is an Optimal Solution.

2.3 FIGURES

Fig. 1

New Optimal AH Method Solving Transportation Problem

Enter No Of Row: 3

Enter No Of Column: 3

Create Matrix

Solve

Computational Procedure of Transportation Model

	D_1	D_2	D_3	Supply
S_1	10	4	11	14
S_2	12	5	8	10
S_3	9	6	7	6
Demand	8	10	12	30

Fig. 2(a)

Proccess

Itration 1.

	D_1	D_2	D_3	Supply
S_1	10	4	11	14
S_2	12	5	8	10
S_3	9	6	7	6
Demand	8	10	12	

	D_1	D_2	D_3	Supply
S_1	10	4	11	14
S_2	9	6	7	6
Demand	8	0	12	

	D_1	D_2	Supply
S_1	10	11	14
S_2	9	7	6
Demand	8	12	

	D_1	D_2	D_3	Supply
S_1	10 (8)	4 (0)	11 (6)	14
S_2	12	5 (10)	8	10
S_3	9	6	7 (6)	6
Demand	8	10	12	
Total	238			

Fig.2(b)

Itration 2.

	D_1	D_2	D_3	Supply
S_1	10	4	11	14
S_2	12	5	8	10
S_3	9	6	7	6
Demand	8	10	12	

	D_1	D_2	D_3	Supply
S_1	10	4	11	14
S_2	12	5	8	10
Demand	8	10	6	

	D_1	D_2	Supply
S_1	10	4	14
S_2	12	5	4
Demand	8	10	

	D_1	D_2	D_3	Supply
S_1	10 (8)	4 (6)	11	14
S_2	12	5 (4)	8 (6)	10
S_3	9	6	7 (6)	6
Demand	8	10	12	
Total	214			

Fig. 2(c)

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Iteration 3.

	D_1	D_2	D_3	Supply
S_1	10	4	11	14
S_2	12	5	8	10
S_3	9	6	7	6
Demand	8	10	12	

	D1	D2	Supply
S_1	10	11	4
S_2	12	8	10
S_3	9	7	6
Demand	8	12	

	D_1	D_2	Supply
S_1	12	8	10
S_2	9	7	6
Demand	4	12	

	D_1	D_2	D_3	Supply
S_1	10 (4)	4 (10)	11	14
S_2	12	5	8 (10)	10
S_3	9 (4)	6	7 (2)	6
Demand	8	10	12	
Total	210			

Fig. 3(a)

Result

Itration 1

	D1	D2	D3	Supply
S1	10 (8)	4 (0)	11 (6)	14
S2	12	5 (10)	8	10
S3	9	6	7 (6)	6
Demand	8	10	12	
Total	238			

C_{21} (12)->C_{22} (5)->C_{12} (4)->C_{11} (10)-> 12-5+4-10 = 1

C_{23} (8)->C_{22} (5)->C_{12} (4)->C_{13} (11)-> 8-5+4-11 = -4

C_{31} (9)->C_{33} (7)->C_{13} (11)->C_{11} (10)-> 9-7+11-10 = 3

C_{32} (6)->C_{12} (4)->C_{13} (11)->C_{33} (7)-> 6-4+11-7 = 6

Itration 2

	D1	D2	D3	Supply
S1	10 (8)	4 (6)	11	14
S2	12	5 (4)	8 (6)	10
S3	9	6	7 (6)	6
Demand	8	10	12	
Total	214			

C_{13} (11) -> C_{12} (4) -> C_{22} (5) -> C_{23} (8) -> 11-4+5-8 = 4

C_{21} (12) -> C_{22} (5) -> C_{12} (4) -> C_{11} (10) -> 12-5+4-10 = 1

C_{31} (9) -> C_{33} (7) -> C_{23} (8) -> C_{22} (5) -> C_{12} (4) -> C_{11} (10) -> 9-7+8-5+4-10 = -1

C_{32} (6)->C_{22} (5)-> C_{23} (8)->C_{33} (7)-> 6-5+8-7 = 2

Fig.3(b)

International Journal of Supply Chain and Logistics
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Vol. 9, Issue No.3, pp 74 - 92, 2025
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Itration 3

	D_1	D_2	D_3	Supply
S_1	10 (4)	4 (10)	11	14
S_2	12	5	8 (10)	10
S_3	9 (4)	6	7 (2)	6
Demand	8	10	12	
Total	210			

C_{13} (11) -> C_{11} (10) -> C_{31} (9) -> C_{33} (7) -> 11-10+9-7 = 3

C_{21} (12) -> C_{23} (8) -> C_{33} (7) -> C_{31} (9) -> 12-8+7-9 = 2

C_{22} (5) -> C_{23} (8) -> C_{33} (7) -> C_{31} (9) -> C_{11} (10) -> C_{12} (4) -> 5-8+7-9+10-4 = 1

C_{32} (6) -> C_{31} (9) -> C_{11} (10) -> C_{12} (4) -> 6-9+10-4 = 3

Solution is Optimal

3. RESULTS AND DISCUSSIONS

In this section, we present the results of applying the newly direct AH optimal method to find the minimized transportation cost for 12 balanced and unbalanced transportation problems (see comparison table no. **10**). The performance of AH method compared with two existing optimal methods: stepping stone method (SSM) and modified distribution method (MODI). Thus the suggested methodology provides optimal solutions for balanced and unbalanced transportation problems, requiring fewer iterations. This means fewer calculations, saving significant time. Our comparative assessment reveals that this proposed method delivers a superior outcome in contrast to other existing heuristic optimal methods. Clearly the results of this study show that the AH approach is an efficient and an effective method for minimizing transportation costs, and it consistently outperforms the above said traditional optimal methods in terms of number of iterations and time consumption.

Moreover, the AH approach was found to be highly efficient and able to find a solution much faster than other methods, even for large supply chains. This is due to the fact that the AH approach considers the maximum range of each supplier, allowing for greater flexibility in the allocation of shipments.

4. SUMMARY OF RESULTS

Comparison Table 10.

Problems	Size	NWCM	LCM	VAM	MODI	AH method
1.	4x4	484	484	476	**412**	412
2.	3x4	1015	1015	779	**743**	743
3.	5x5	68969	68969	68804	**59356**	59356
4.	3x5	143	143	91	**80**	80
5.	3x4	670	670	650	**610**	610
6.	3x4	3680	3680	3520	**3460**	3460
7.	4x3	102	102	80	**76**	76
8.	3x4	726	726	542	**506**	506
9.	3x4	273	273	204	**200**	200
10.	3x3	160	160	150	**148**	148
11.	4x4	486	486	225	**180**	180
12.	3x5	308	308	175	**172**	172

5. CONCLUSION

In this paper, we proposed a new direct AH optimal method to find minimized transportation cost for balanced and unbalanced transportation problems. A software package based on Java script programming developed for AH method. Hundreds of balanced and unbalanced problems of different sizes solved by using the software which provided optimal solutions. We compared proposed method with two existing optimal methods, namely the stepping stone method and the MODI method, and showed that AH optimal method is a direct method that consumes less time and contains fewer steps than the other methods. We also tested proposed method on particular 12 different problems taken from research journals and proved that it always yields an optimal solution that matches the solutions obtained by the other optimal methods (see comparison table no. **10**). Proposed method has important applications in real life, such as logistics, supply chain management, network design, etc., where minimizing transportation cost is a crucial objective.

For future work, we suggest to extend new optimal method to handle more complex transportation problems, such as those involving multiple modes of transportation, stochastic demand and supply, or environmental factors. We conclude that new method AH is a simple, efficient, and reliable way to solve transportation problems and optimize transportation cost.

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Availability of data

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Conflict of interest

The authors declare there is no competing interests (conflicts of interest).

Authors contributions

Authors contributions included

7. ETHICS DECLARATIONS

Ethical approval and consent to participate

Not Applicable

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